Wavelet Packet Spectrum for Analysis of Texture using Dual Tree Complex Wavelet Transform

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Abstract—Two Dimensional Wavelet Packet Spectrum estimator is derived from statistical properties of wavelet packet coefficients of random process using Dual tree complex wavelet transform. Previously Wavelet Packet Spectrum is derived using Shannon wavelet but it is unable to perform orthogonal analysis. The results illustrate the effectiveness of the wavelet-based spectrum estimation using Dual tree complex wavelet transform that it supports good comprise of smoothness regarding Hurst parameter .

Index Terms— 2-D Wavelet Packet Spectrum (WPS), random fields, spectral analysis, spectrum estimation, Dual Tree Complex Wavelet, texture.

1. INTRODUCTION

In the visual world, textures can be regarded as the visual appearances of surfaces and may be perceived as being directional or non-directional, smooth or rough, coarse or fine, regular or irregular, etc. Several textures are observed on both artificial and natural objects and scenes. The surface characteristics of textures can be used to recognize objects in image, to segment image and to understand an image [1]. The Textures play an important role, computer vision and pattern recognition methods. However, illumination and environment conditions can affect the appearance of textures, and complicate the tasks. Textures in real images can vary in scale, brightness, and rotation as imaging conditions change. Therefore, to enable texture analysis in real images, texture representation should be invariant to imaging conditions such as non-rigid deformation, viewpoint, scaling and lighting. A brief review of the invariant texture analysis methods is presented in [2]. The idea behind the definition of 2-D wavelet spectra is the following: since the tensor-product wavelet multi-resolution analysis of d dimensional data comprises of 2^d -1 detailed spaces, with these each space containing the hierarchy of subspaces with nested dyadic resolutions, it is quite natural to assess the energy scaling in each hierarchy. This leads to 2^d -1 concurrent power spectra describing a single ddimensional data set. For example, multi-resolution analysis of images leads to three detail spaces described as "horizontal", "vertical" or "diagonal," depending on the selection of the decomposing 2-D wavelet, or the order of applications of high- and low-pass wavelet filters on the rows and columns of 2-Dimensional objects. Each of these three directional detailed spaces contains a nested hierarchy of sub-matrices corresponding to image details at different scales and each leads to a distinctive power spectra.

2. DUAL TREE COMPLEX WAVELET TRANSFORM

The complex wavelet transform (CWT) is a complex-valued extension to the standard discrete wavelet transform (DWT). It is a 2-Dimensional wavelet transform which provides multi-resolution, sparse representation, and useful characterization of the structure of image. The Dual-tree complex wavelet transform (DTCWT) calculates the complex transform of a signal using two separate DWT decompositions (tree b and tree at is possible for one DWT to produce the real coefficients and the imaginary coefficients. This redundancy of these two provides extra information for analysis but at the expense of extra computational power It also provides approximate shiftinvariance (unlike the DWT) yet still allows perfect reconstruction of the signal. The filters design is particularly important for the transform to occur correctly and the necessary characteristics are:

- 1. The low-pass filters in the two trees must differ by half a sample period
- 2. Reconstruction filters for reverse analysis
- 3. Tree b filters are the reverse of tree a filters
- 4. Both trees have same frequency response

3. WAVELET PACKET SPECTRUM ESTIMATOR *A. 2-D Wavelet Packet*

We consider the 2-D separable wavelet packet decomposition in a continuous time signal setting for presenting theoretical results [8]. Advanced concepts and algorithms concerning 1D and 2-D wavelet packet analysis can be found in [8]. The reader is also invited to refer to [9], [10] (wavelets) and [6], [11] (wavelet packets) for more details on the statistical properties of wavelet transforms, when the decomposition relates to a random process. In this

decomposition, the wavelet paraunitary filters H_0 (low-pass, scaling filter) and H_1 (high-pass, wavelet filter) are used to split the input functional space $\mathbf{U} = \mathbf{W}_{0,0} \subset L^2(\mathbb{R}^2)$ into orthogonal subspaces.

Assume that the scaling filter is with order r: $H_0 \equiv Hr^0$, where r is the largest non-negative integer

$$\operatorname{Hr}^{0}(\omega) = \left(\frac{1+e^{-i\omega}}{2}\right)^{r} Q(e^{i\omega}) \tag{1}$$

filter H_0^{s} denoting the scaling filter associated with the Symmlet wavelet. Then the 1D multiscale filters

 $(\tilde{H}^r j, ni)i=1,2$ have very tight supports when r is large.

B. Wavelet Packet Paths

This section presents a specific wavelet packet path description derived from the binary sequence approach of [13] for representing nested wavelet packet subspaces. This description is suitable for establishing asymptotic properties of 2-D wavelet packets with respect to the increase of the decomposition level. It is worth mentioning that some specific paths will present singular behavior: The wavelet coefficients of certain non-stationary random fields on the sub bands associated with these singular paths will remain nonstationary

As a matter of example,

- 1) The isotropic Fractional Brownian field analyzed which admits a unique singular path: the approximation path denoted by P_0 and associated with frequency indices nP_0 (j) = 0 for every j.
- 2) The separable Fractional Brownian field analyzed which admits frequency indices n(j) such that $n_1(j) = 0$ (respectively $n_2(j) = 0$) for every j as singular frequency indices. The set of (singular) paths associated with these frequency indices will be denoted by $Pn/n_1=0$ (resp. $Pn/n_2=0$).

In order to select the best performance of wavelet-based estimator, we simulated 1000 fractional Brownian fields with various H and for each field estimated the Hurst parameter in each of the three directions.

Table 1: Means and standard deviations (in brackets) of the estimated

Hurst exponents, by the wavelet-based estimators (D4, S4, C1

and Haar) evaluated on 1000 simulated random fields with H =

0:4 and length $n = 256 \times 256$, with and without noise in each case.

Wavelets	H=0.4		
	Snr=∞	Snr=20	
D4	0.3920 (0.042)	0.3828 (0.325)	

S4	0.3968 (0.042)	0.3838 (0.326)
C1	0.3865 (0.042)	0.3766 (0.041)
Haar	0.3508 (0.041)	0.3427 (0.040)

The wavelet based estimator was more robust when the data are contaminated by noise, even at a low level.

For comparative purpose we use Symmlet 4 since this filter provides a good compromise of smoothness, locality and near-symmetry in Table 1 we provide the summary of this experiment.

C. 2-D Wavelet Packet based Spectrum Estimation

The Wavelet Packet Spectrum is estimated by using Symmlet wavelet. The spectrum estimation method presented in this section follows from the asymptotic analysis of the autocorrelation functions of the 2-D wavelet packet coefficients. This asymptotic analysis is performed with respect to the wavelet order r and the wavelet decomposition level j. When r increases, the asymptotic behavior of the sequence of wavelet functions is driven by the Symmlet wavelet functions. In this respect, we consider the Symmlet wavelets in below and derive asymptotic results with respect to the wavelet decomposition level.

The following provides a non-parametric method for estimating spectrum γ of 2-D random fields on the basis of the convergence criterion. It follows the equation

 γ ($\omega_1[P], \omega_2[P]$) = lim $j \rightarrow +\infty R^s _{j,n}[0, 0]$ so that the continuity points of spectrum γ can be estimated by sub band variances (values{ $R^s _{j,n}[0, 0]$ } provided that the Symmlet wavelet is used and j is large enough. Furthermore, we can derive from the convergence criteria, several spectrum estimators by considering wavelets with finite orders r, the accuracy of the spectrum estimation being dependent on the wavelet order as shown in Proposition 3 below. Assuming a uniform sampling (regularly spaced frequency plane tiling), the method applies upon the following steps.

- 1) Define a frequency grid compose with frequency points $(\frac{P_1\pi}{2i}, \frac{P_2\pi}{2^i})$ for $P_1, P_2 \in \{0, 1, \dots, 2^j 1\}$ (natural ordering).
- 2) Compute, the index n ∈ {0, 1, ..., 4^j − 1} (corresponding to the wavelet packet ordering) associated with (P₁, P₂).
- 3) Set, for any pair (P₁, P₂) given in step 1) and the corresponding n obtained from second step).

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There are three propositions to find Wavelet based Spectrum as shown below:

Proposition 1: The height of this fractional moment depends on the scaling function associated with the wavelet packet decomposition. Note also that when both n1 = n2 = 0, the non-stationary in wavelet coefficients is more intricate, mainly because the analyzing function has no vanishing moments.

Proposition 2: The autocorrelation function of the wavelet packet coefficients of separable and isotropic Fractional Brownian Field can be written in the integral form.

Proposition 3: We derive that the bias of the estimator given by depends on the decomposition level and wavelet order used. This bias tends to 0 when both j and r tends to infinity.

4. FABRIC TEXTURE ANALYSIS USING WAVELET PACKET SPECTRUM

The below shown Figure 1 is one of the Fabric texture and then



Figure 1: Fabric texture

The Wavelet Packet Spectrum estimator is applied to this texture information to derive Wavelet Packet Spectrum of this texture which is as shown below Figure 2:



Figure 2: Wavelet Packet Spectrum for above fabric texture

The comparison should be made with Wavelet Packet Spectrum of Symmlet with Shannon which is shown in Figure 3.



Figure 3: Textures image and their spectra γ computed by using

Discrete Wavelet Packet Transforms using Symmlet and

Shannon. Abscissa of the spectra images consists regular grid $[0, \pi/2] \times [0, \pi/2]$.

Note: Colors represented in Figure 3 are simulated from a light source in order to ease 3-D visualization: red color [value 1] corresponds to fully illuminated shapes whereas blue color [value 0] is associated to shaded areas, green color corresponds to value 0.5.

5. ADVANTAGES

- 1. The best basis can provides suitable frequency sub bands for the signal representation.
- 2. The over complete structure of WPT provides flexibility for the signal representation to achieve better classification accuracy.
- The subject-based adaptation feature extraction with this method constructs a wavelet packet best basis fitted for each object and so it can find the suitable and specific features for a subject's signals.
- 4. Provides more accurate information without losing single unit also

6. APPLICATIONS

- 1. Spread-spectrum image watermarking.
- 2. Hurst parameter estimation for self-similar medical images, see for instance [6].
- 3. Texture modeling by using Wold decompositions estimation the poles of the spectrum is necessary to determine the spectral singularities involved in the deterministic texture contribution. These poles are associated with peaks of the spectrum and their number, as well as their location determines the accuracy of the modeling.

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7. CONCLUSION

This section provides experimental results on spectral analysis of textures. A Wavelet Packet Spectrum of fabric texture image is provided in Figure.2. The Wavelet Packet Spectra have been computed with the decomposition level is 6 and the Symmlet wavelet with order r = 7 is used. Spectra computed from the Wavelet Packet Spectrum using Shannon wavelet are also given in this figure 3, for comparison purpose.

From a visual analysis of images, one can remark that most of this texture exhibit non overlapping textons replicating repeatedly: thus, coarsely, we can distinguish several frequencies having significant variance contributions (from a theoretical consideration), when the texture does not reduce to the replications of a single texton. In addition, when these textons occupy approximately the same spatial area (see for instance "Fabric" textures in Figure 1), the frequencies with high variance contributions (peak in the spectrum) are close in terms of their spatial location (from a theoretical consideration).

The above heuristics, issued from visual image analysis, are confirmed by considering the Wavelet Packet Spectrum (see for instance spectra of "Fabric" textures in Figure 2), whereas, in most cases, the two dimensional discrete Fourier transform exhibits only one peak.

On comparison with Shannon wavelet, Symmlet wavelet provides a good compromise of smoothness, locality and near-symmetry. It does not lose even single unit of information.

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